A1 Expressions and Formulae

Knowledge Organiser

Keywords

Rearrange - use *inverse operations* to change the subject

Subject - the variable that is isolated on one side of the formula

Quadratic equation - An equation where the highest power is a square (x^2).

Expand - multiply out the brackets

Factorise - putting *into* brackets (factors)

Solve - find the value of the unknown (x)

Identity - equation that is true no matter what values are used. ≡ means 'equivalent to'

Formulae

Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completed square form

$$(x + a)^2 + b$$

Factorised form

$$(x \pm a)(x \pm b)$$

Examples – Solving Quadratics

Factorising and Solving Quadratics

Factorise $x^2 + 2x - 15$

Factor pairs of 15 are 1 and 15

3 and 5

We choose the factor pair 3 and 5 as these can be used to make 2 by addition or subtraction.

$(x + 5)(x - 3)$

You can solve quadratics if they are '= 0'

e.g. $(x + 5)(x - 3) = 0$

gives solutions $x = -5$ or $x = 3$

D.O.T.S

Fully factorise

$25x^2 - 16$ Square root both terms $(5x + 4)(5x - 4)$

Key Facts/Examples

Changing the Subject

Make y the subject

$$2x + 3y = 12$$

$$\xrightarrow{-2x}\quad 3y = 12 - 2x$$

$$\xrightarrow{\div 3}\quad y = 4 - \frac{2}{3}x$$

Make r the subject

$$V = \frac{4}{3}\pi r^3$$

$$\xrightarrow{\times 3}\quad 3V = 4\pi r^3$$

$$\xrightarrow{\div 4\pi}\quad \frac{3V}{4\pi} = r^3$$

$$\xrightarrow{\sqrt[3]{}}\quad \sqrt[3]{\frac{3V}{4\pi}} = r$$

Expand Double/Triple Brackets

Expand and simplify $(x - 2)(2x + 3)(x + 1)$

	$2x$	$+3$
x	$2x^2$	$+3x$
-2	$-4x$	-6

$2x^2 - x - 6$

	$2x^2$	$-x$	-6
x	$2x^3$	$-x^2$	$-6x$
$+1$	$+2x^2$	$-x$	-6

$2x^3 + x^2 - 7x - 6$

Quadratic Formula

$$2x^2 + 5x + 1 = 0$$

$a = 2 \quad b = 5 \quad c = 1$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 1}}{2 \times 2}$$

$$x = \frac{-5 \pm \sqrt{17}}{4}$$

$$x = \frac{-5 + \sqrt{17}}{4} \qquad x = \frac{-5 - \sqrt{17}}{4}$$

Completing the Square

$$x^2 + 4x - 3$$

$$(x + 2)^2 - 4 - 3$$

$$(x + 2)^2 - 7$$

...and to solve...

$$(x + 2)^2 - 7 = 0$$

$$(x + 2)^2 = 7$$

$$x + 2 = \pm\sqrt{7}$$

$$x = -2 \pm \sqrt{7}$$

A1 Expressions and Formulae

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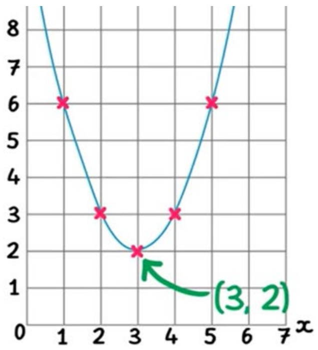
Turning Point/Vertex

Opposite sign of number within bracket

Final term

$y = (x - 3)^2 + 2$

Vertex = (3 , 2)



Identities

Formula	A fact or a rule that **relates two or more** quantities using mathematical symbols. A formula contains an equals sign '='. E.g. $D = S \times T$
Equation	A statement showing that **two mathematical expressions are equal to one another**. An equation contains an equals sign '='. $5 - 3x = 2x + 4$
Expression	A mathematical statement combining **numbers, symbols and operators**. An expression does **not** contain an equals sign. $2x^2 + 4x - 7$
Identity	A relationship showing that **two mathematical statements are identical**. An identity contains a **triple bar** (tribar) sign '≡'. $2(x+1) \equiv 2x + 2$

Algebraic Fractions

Simplify fully $\dfrac{12m + 18}{2m^2 + 3m}$

$$= \frac{6\cancel{(2m+3)}}{m\cancel{(2m+3)}}$$

$$= \frac{6}{m}$$

Simplify fully $\dfrac{9y^2 - 1}{3y^2 + 19y + 6}$

$$= \frac{\cancel{(3y+1)}(3y-1)}{\cancel{(3y+1)}(y+6)}$$

$$= \frac{3y - 1}{y + 6}$$

Algebraic Proof

Prove that $(n + 2)(n - 2) + n(n + 6)$ is even

$$(n+2)(n-2) + n(n+6) = n^2 - 2n + 2n - 4 + n^2 + 6n$$
$$= 2n^2 + 6n - 4$$
$$= 2(n^2 + 3n - 2)$$

$(n^2 + 3n - 2)$ is an integer so $2(n^2 + 3n - 2)$ is a multiple of 2

Prove that $(n + 4)^2 - (n - 2)^2$ is a multiple of 12

$$(n+4)^2 - (n-2)^2 = (n+4)(n+4) - (n-2)(n-2)$$
$$= n^2 + 4n + 4n + 16 - (n^2 - 2n - 2n + 4)$$
$$= n^2 + 8n + 16 - n^2 + 2n + 2n - 4$$
$$= 12n + 12$$
$$= 12(n + 1)$$

integer

$(n + 1)$ is an integer so $12(n + 1)$ is a multiple of 12.